Estimation of Intra-operative Brain Shift Based on Constrained Kalman Filter

Abstract

In this study, the problem of estimation of brain shift is addressed by which the accuracy of neuronavigation systems can be improved. To this end, the actual brain shift is considered as a Gaussian random vector with a known mean and an unknown covariance. Then, brain surface imaging is employed together with solutions of linear elastic model and the best estimation is found using constrained Kalman filter (CKF). Moreover, a recursive method (RCKF) is presented, the computational cost of which in the operating room is significantly lower than CKF, because it is not required to compute inverse of any large matrix. Finally, the theory is verified by the simulation results, which show the superiority of the proposed method as compared to existing method in terms of accuracy and efficiency.

Keywords: Brain shift, constrained Kalman filter, neuronavigation systems, estimation theory.

1. Introduction

Image guided neurosurgery (IGNS) provides the exact position of surgical tools in the patient's body. This property is employed in computer-aided surgery and results in a more precise treatment. The accuracy of IGNS is largely dependent on the used images. Furthermore, due to the brain tissue flexibility, it deforms after dural opening; thus, pre-operative images are not valid anymore, which result in a less accurate navigation. This brain deformation is known as "brain shift" in which many factors are involved such as tumor resection, gravity, pharmacologic responses, loss of cerebrospinal fluid, etc. Another problem is that the precise impact of each of these factors is not specified [1, 2]. There are mainly two solutions for the problem of brain shift: intra-operative medical imaging and biomechanical models of the brain. Intra-operative MRI (iMRI) [3], intra-operative CT (iCT) [4], and intra-operative Ultrasound (iUS) [5] fall into the first category. iMRI is time-consuming and requires expensive non-magnetic tools. iCT can be harmful to health in a long time due to high dose X-ray used in it. iUS is much cheaper than iMRI, but it generally results in lower quality images. The later approach uses physical features of brain tissues that are expressed by partial differential equations (PDE). Afterwards, the PDE is solved using boundary conditions and the solution of which is used to improve pre-operative images. In addition, it results in high resolution images as a result of using pre-operative ones, on which this paper is also based.

Several models have been presented for assessing the biomechanical behaviour of brain tissues. These models consist of mass spring model [6], linear elastic model [7], nonlinear model [8], and mechanical model based on consolidation theory [9]. Mass-spring model is over-simplified and has low accuracy. In [10], it is shown that with using an appropriate finite deformation solution, the choice of linear elastic and nonlinear models do not affect the accuracy. As a result, the linear elastic model is recommended due to its reduced computations. In [11], a comparison is performed between the mechanical model and linear elastic model, that shows the elastic model is more accurate.

The measurement of brain surface displacement is employed in [12, 13, 14, 15, 16, 17] with somewhat promising results. In [12], a method is presented to improve solution of linear elastic model using image processing theory, the main drawback of which is dependency on view angle. In [13], an atlas-based method is proposed and its sensitivity analysis is studied in [14]. Optimizations are also carried out based on calculus of variation [15], steepest gradient descent [16], and game theory [17]. The defect of the above mentioned approaches is that only brain surface estimation error is minimized, but subsurface tissues estimation is not investigated.

One of the well-known approaches in estimation problems is Kalman filter which is utilized for parameter estimation of dual-rate systems [18] and MicroElectroMechanical systems [19]. If there are constraints on estimation problem, constrained Kalman filter (CKF) will result in a more accurate estimation [20, 21], which is employed in [22, 23, 24]. In [22], state estimation problem of power systems is addressed. Two algorithms for estimation with inequality constraint are derived in [23], and a reduced-order Kalman filter is presented in [24].

In this paper, linear elastic model is adopted and the desired estimation is calculated using the CKF. To this end, brain shift is considered as a Gaussian random vector with a known mean and an unknown covariance. Then, the best estimation is found such that an upper bound of the estimation error variance will be minimized. Furthermore, a recursive method (RCKF) is presented to prevent computing inverse of a large matrix which results in a less computational cost in the operating room. Moreover, superiorities of the proposed approach over the existing methods is illustrated using finite element method. The claims are also demonstrated by simulation results.

2. Model and Assumptions

According to the introduction, the linear elastic model is utilized as governing equations of the brain in this paper. This model with corresponding assumptions for estimation are explained in this section.

2.1. Linear elastic model

The linear elastic model considers brain as a linear elastic continuum with no initial stresses or strains. The energy of brain deformation due to the external forces can be expressed as [25]

$$E = \frac{1}{2} \int_{\Omega} \sigma^T \varepsilon d\Omega + \int_{\Omega} F^T u d\Omega \tag{1}$$

where F is the total external force applied to the body, Ω is the elastic body, u is the displacement vector, ε is the stress vector, and σ is the strain vector. The relationships between stress, strain and displacement vector are as follows

$$\sigma = D\varepsilon,$$
$$\varepsilon = Lu$$

where L is an operational matrix and D is the elasticity matrix that describes properties of the body [25]. According to the *principle of minimum potential energy*[26], only the actual value of u can minimize (1) for a certain body. Hence, any estimation of u is the suboptimal solution of minimizing (1). If finite element method (FEM) be used to minimize (1), volume of the brain will be discretized and yields the following equation for brain model [27]

$$Kx = b \tag{2}$$

where $b \in \mathbb{R}^n$ includes boundary conditions and surface forces, $x \in \mathbb{R}^n$ expresses discrete quantities of u in the FEM's nodes, and $K \in \mathbb{R}^{n\times n}$ is the stiffness matrix that has the role of discrete equations. Unfortunately, measurement of b is not possible in the operation room and consequently it is not possible to calculate x from (2) directly. As mentioned in the introduction, brain surface imaging can be utilized to improve brain shift estimation. Since x expresses position of all nodes in the brain, a full row rank matrix C can be found such that the position of brain surface is calculated from x. Then it can be assumed that

$$y = Cx \tag{3}$$

where $y \in \mathbb{R}^m$ $(m < n)$ is the measurement of brain surface. Therefore, it is not possible to calculate x from y directly.

2.2. Assumptions

The following assumptions are made to solve the problem.

(i) $\text{rank}(K) = n$.
(ii) An initial estimation $\bar{b}$ of b is available.

It is noted that none of the assumptions is restricting. The first assumption is held due to (2) is obtained from a valid FEM and proper boundary conditions [28]. The second assumption states that an initial estimation of b is available, and to this end physics of the problem can be used. For instance, it can be assumed that $\bar{b}$ is due to the gravity and loss of cerebrospinal fluid[29].

3. Brain Shift Estimation Using Constrained Kalman Filter

To estimate brain shift by CKF, the vector b is considered as a Gaussian random vector with mean vector $\bar{b}$ and unknown covariance matrix. Therefore, x is also a Gaussian random vector with unknown covariance matrix, and its mean vector $(\bar{x})$ using (2) can be given by

$$\bar{x} = K^{-1}\bar{b}. \tag{4}$$

One way to compute an estimation of x $(\hat{x})$ is to estimate b, then $\hat{x}$ can be computed from (2) as

$$\hat{x} = K^{-1}\hat{b} \tag{5}$$

where $\hat{x}$ and $\hat{b}$ are estimates of x and b, respectively. This approach is known as *inverse method* [29, 16]. The purpose of estimation $\hat{b}$ is to minimize variance of estimation error of b; therefore, the following cost function is considered

$$J = \mathrm{E}\left[(b-\hat{b})^T(b-\hat{b})\right]. \tag{6}$$

If the cost function is minimized without any constraint, $\hat{b}$ will be found identical to $\bar{b}$. To improve the estimation, it is needed to define a well-suitable constraint. By substitution of (2) into (3), one can get

$$y = CK^{-1}b.$$

It is obvious that the estimation of b should satisfy this equation. Consequently, the following constraint can be considered

$$y = CK^{-1}\hat{b}. \tag{7}$$

Now $\hat{b}$ should minimize the cost function (6) subject to the constraint (7). To solve the problem, the constraint can be adjoined to the cost function using Lagrange multipliers; therefore, the resulting augmented cost function can be given as

$$J_a = \mathrm{E}\left[(b-\hat{b})^T(b-\hat{b})\right] + 2\lambda^T\left(y - CK^{-1}\hat{b}\right).$$

The expanded form of J_a is

$$J_a = \int b^T b f(b)\ db - 2\hat{b}^T \int b f(b)\ db \\ + \hat{b}^T\hat{b}\int f(b)\ db + 2\lambda^T\left(y - CK^{-1}\hat{b}\right).$$

The integral of the second term is the mean of b, i.e. $\bar{b}$. Moreover, by considering the properties of probability density function, the integral of the third term is equal to one; therefore, the final equation can be expressed as

$$J_a = \int b^T b f(b)\ db - 2\hat{b}^T\bar{b} + \hat{b}^T\hat{b} \\ + 2\lambda^T\left(y - CK^{-1}\hat{b}\right). \tag{8}$$

To minimize (8), the following equations must hold

$$\frac{\partial J_a}{\partial \hat{b}} = -2\bar{b} + 2\hat{b} - 2K^{-T}C^T\lambda = 0, \tag{9}$$

$$\frac{\partial J_a}{\partial \lambda} = 2\left(y - CK^{-1}\hat{b}\right) = 0. \tag{10}$$

By pre-multiplication of CK^{-1} in (9) and substituting $CK^{-1}\hat{b}$ from (10), one can get

$$CK^{-1}K^{-T}C^T\lambda = y - CK^{-1}\bar{b}.$$

Now by inverting the matrix $CK^{-1}K^{-T}C^T$ and substituting λ into (9), we can get the estimation as

$$\hat{b} = \bar{b} + K^{-T}C^T(CK^{-1}K^{-T}C^T)^{-1}(y - CK^{-1}\bar{b}). \tag{11}$$

Once $\hat{b}$ is obtained, $\hat{x}$ can be found from (4) and (5)

$$\hat{x} = \bar{x} + K^{-1}K^{-T}C^T(CK^{-1}K^{-T}C^T)^{-1} \times \left(y - CK^{-1}\bar{b}\right). \tag{12}$$

The above estimated $\hat{b}$ minimizes (8), however the question to be answered is how well x is estimated. The following theorem, which manifests one of the contribution of the paper, answers that question.

Theorem 1. *Consider equations* (2), (3) *and* (4). *If the general estimation* $\hat{x}_L$ *for a gain matrix* L *is given by*

$$\hat{x}_L = \bar{x} + K^{-1}L\left(y - CK^{-1}\bar{b}\right), \tag{13}$$

and the estimation error is considered as

$$e_L = x - \hat{x}_L, \tag{14}$$

then

1. *The general estimation is unbiased, i.e.* $\mathrm{E}[e_L] = 0$.
2. *The best estimation that minimizes* $\max(\mathrm{Var}(e_L))$ *in the case of unknown* $\mathrm{Cov}(b)$ *is equal to* $\hat{x}$ *given by* (12).
3. *If the estimation is computed recursively, as*

$$\begin{aligned}\hat{x}_{k+1} &= \hat{x}_k + K^{-1}K^{-T}C^T(CK^{-1}K^{-T}C^T)^{-1} \\ &\quad \times (y - C\hat{x}_k), \\ \hat{x}_0 &= \bar{x},\end{aligned} \tag{15}$$

   then

$$\hat{x}_{k+1} = \hat{x}_1, \qquad k \geq 1.$$

Proof.

I) By substitution (3) into (13), one can get

$$\hat{x}_L = \bar{x} + K^{-1}L\left(Cx - CK^{-1}\bar{b}\right).$$

Now by using (4), mean of $\hat{x}_L$ can be easily calculated as

$$\mathrm{E}\left[\hat{x}_L\right] = \bar{x}.$$

II) To prove the second property, it is needed to calculate the covariances of b and x. It is valid to assume that covariance of b is a positive definite but unknown matrix. Then we can write

$$\begin{aligned} \mathrm{E}\left[(b-\bar{b})(b-\bar{b})^T\right] &= Q, \\ \mathrm{E}\left[(x-\bar{x})(x-\bar{x})^T\right] &= K^{-1}QK^{-T} \end{aligned} \tag{16}$$

where $Q = Q^T > 0$ is unknown. Using (13), (14), and (16), the covariance of e_L can be obtained as follows

$$\mathrm{Cov}(e_L) = \left(I - K^{-1}LC\right)K^{-1}QK^{-T} \times \left(I - K^{-1}LC\right)^T. \tag{17}$$

The variance of e_L is equal to the trace of the covariance matrix

$$\mathrm{Var}(e_L) = \mathrm{Tr}\{\mathrm{Cov}(e_L)\}, \tag{18}$$

also the following row form can be considered

$$\left(I - K^{-1}LC\right)K^{-1} = \begin{bmatrix} l_1^T \\ l_2^T \\ \vdots \\ l_n^T \end{bmatrix}. \tag{19}$$

By using (17) and (19), (18) can be written as

$$\mathrm{Var}(e_L) = \sum_{i=1}^{n} l_i^T Q l_i. \tag{20}$$

In general, the following equation holds for quadratic form $l_i^T Q l_i$ for a symmetric positive definite matrix Q

$$\lambda_{\min}(Q) l_i^T l_i \le l_i^T Q l_i \le \lambda_{\max}(Q) l_i^T l_i. \tag{21}$$

By using (21), the following inequality can be obtained from (20)

$$\lambda_{\min}(Q)\sum_{i=1}^{n} l_i^T l_i \leq \mathrm{Var}(e_L)$$
$$\leq \lambda_{\max}(Q)\sum_{i=1}^{n} l_i^T l_i. \quad (22)$$

All eigenvalues of Q are positive and unknown, since it is an unknown positive definite matrix. Therefore, to achieve the best estimation, it is only possible to minimize the coefficient of the eigenvalues in (22). Consequently, L should minimize the following cost function, which is obtained from (19) and (22)

$$J_{\mathrm{Var}} = \mathrm{Tr}\{\left(I - K^{-1}LC\right) K^{-1}$$
$$\times K^{-T} \left(I - K^{-1}LC\right)^T\}. \quad (23)$$

Optimal matrix L that minimizes (23) can be calculated from $\frac{\partial J_{\mathrm{Var}}}{\partial L} = 0$, and it is

$$L = K^{-T}C^T \left(CK^{-1}K^{-T}C^T\right)^{-1}.$$

It is obvious that by substituting L into (13), $\hat{x}_L$ will be obtained equal to (12).

III) To prove the third property, we use induction. For $k = 1$, we can write

$$\hat{x}_2 = \hat{x}_1 + K^{-1}K^{-T}C^T(CK^{-1}K^{-T}C^T)^{-1}$$
$$\times (y - C\hat{x}_1) \quad (24)$$

and by substituting $\hat{x}_1$ form (15) into (24), it can be easily shown that $\hat{x}_2 = \hat{x}_1$. Then, it can be shown in a similar way that if (15) holds for any k, it holds for $k + 1$.

□

The first and second properties in the Theorem 1 state that the given estimation (12) is unbiased and has the best variance of estimation error under unknown covariance of b. The third property shows that using the recursive form of (15) does not increase the accuracy of estimation, then one time computation of (12) is sufficient.

4. Brain Shift Estimation Using a Recursive Form of Constrained Kalman Filter

In addition to the accuracy, estimation of brain shift should be fast due to its application in the operating room. Furthermore, the proposed CKF method in the previous section is directly dependent on the inverse of K, which in practice is a very large matrix; therefore, it is computationally expensive. To solve this drawback, in this section a recursive form of CKF method (RCKF), which dose not use K^{-1}, is proposed.

To avoid computing K^{-1}, one can use the decomposition $K = S - T$, in which S is an invertible matrix [30]. Consequently, (2) and (3) can be written as

$$\begin{aligned} x &= S^{-1}Tx + S^{-1}b, \\ y &= Cx. \end{aligned} \tag{25}$$

To estimate x, a recursive equation can be utilized as

$$\hat{x}_{k+1} = S^{-1}T\hat{x}_k + S^{-1}\tilde{b}, \tag{26}$$

where $\hat{x}_k$ and $\tilde{b}$ are the estimation of x and b respectively. The goal of estimation is to find $\tilde{b}$ so that mean and variance of estimation error $e_k = x - x_k$, tends to zero and be minimized, respectively. The dynamic equation of the error can be expressed as

$$e_{k+1} = S^{-1}Te_k + S^{-1}\left(b - \tilde{b}\right); \tag{27}$$

therefore, to have $\lim_{k\to\infty} \mathrm{E}\left[e_k\right] = 0$, we must have

1. $\mathrm{E}[b] = \mathrm{E}[\tilde{b}]$,
2. All eigenvalues of $S^{-1}T$ are located strictly inside the unit circle.

If $\tilde{b}$ is considered as

$$\tilde{b} = \bar{b} + L\left(y - CK^{-1}\bar{b}\right), \tag{28}$$

where L is an arbitrary matrix, the first condition is held. To satisfy the second condition, the following lemma is presented.

Lemma 1. *Let $K = S - T$, then there is a matrix S so that all eigenvalues of $S^{-1}T$ are located strictly inside the unit circle, if and only if*

$$\mathrm{rank(K) = n},$$

and to find the matrix S, a pole placement problem needs to be solved.

Proof. Define $K = S - T$. Then, the matrix $S^{-1}T$ can be written as

$$S^{-1}T = I - S^{-1}K. \tag{29}$$

Consequently, the matrix S^{-1} leads to eigenvalue placement of the pair (I, K). The remaining of the proof is divided into two parts:

1. what condition should be satisfied for existence of P such that all eigenvalues of $I - PK$ are strictly inside the unite circle?
2. If such matrix exists, is that invertible?

The matrix P exists if and only if the pair (I, K) is observable. Observability matrix is

$$\phi_o = \begin{bmatrix} K^T & K^T I & K^T I^2 & \dots & K^T I^{n-1} \end{bmatrix}^T,$$

and obviously

$$\mathrm{rank(K) = n \Longleftrightarrow rank(\phi_o) = n.}$$

To prove the second part, we need to show that the matrix P is full rank if it does exist. To this end, the Jordan form of $I - PK$ can be written as

$$I - PK = M\Lambda M^{-1},$$

where matrix Λ is a diagonal matrix with the diagonal elements which are strictly inside the unite circle and M is the modal matrix. Pre- and post-multiply the above equation by M^{-1} and M respectively, one can get

$$M^{-1}PKM = I - \Lambda. \tag{30}$$

Since all diagonal elements of matrix Λ are in the unite circle, the right hand side of the above equation is full rank; thus using this fact and Sylvester's law of degeneracy [31], it can be concluded that all matrices in the left hand side involving P are full rank. $\square$

Considering the obtained conditions for computing an unbiased estimation, it is needed to find another conditions to minimize the variance of estimation error. To this end, the recursive equation of (27) can be written as

$$e_k = \left(S^{-1}T\right)^k e_0 + \sum_{i=1}^{k} \left(S^{-1}T\right)^{i-1} S^{-1} \left(b - \tilde{b}\right),$$

then covariance of e_k can be computed as

$$\begin{aligned}\mathrm{Cov}(e_k) &= \left(S^{-1}T\right)^k \mathrm{E}\left[e_0 e_0^T\right]\left\{(S^{-1}T)^k\right\}^T \\ &+ \left(S^{-1}T\right)^k \sum_{j=1}^{k} \mathrm{E}\left[e_0(b-\tilde{b})^T\right] S^{-T}\left\{(S^{-1}T)^{j-1}\right\}^T \\ &+ \sum_{i=1}^{k} \left(S^{-1}T\right)^{i-1} S^{-1}\mathrm{E}\left[(b-\tilde{b})e_0^T\right]\left\{(S^{-1}T)^k\right\}^T \\ &+ \sum_{i=1}^{k}\sum_{j=1}^{k} \left(S^{-1}T\right)^{i-1} S^{-1}\mathrm{E}\left[(b-\tilde{b})(b-\tilde{b})^T\right] \\ &\qquad \times S^{-T}\left\{(S^{-1}T)^{j-1}\right\}^T .\end{aligned} \tag{31}$$

Considering this fact that unknown variable is constant, it is desired to find $\tilde{b}$ such that minimizes the variance of e_k when k goes to infinity.

According to the Lemma 1, all eigenvalues of $S^{-1}T$ are strictly inside the unite circle, hence, we can write

$$\lim_{k\to\infty} \lambda_i^k = 0,$$
$$\lim_{k\to\infty} \sum_{j=1}^{k} \lambda_i^{j-1} = \frac{1}{1-\lambda_i}$$

where λ_i is the i-th eigenvalue of the matrix $S^{-1}T$; therefore, using Caley-Hamilton theorem [31], the following equations can be stated

$$\begin{aligned}\lim_{k\to\infty}\left(S^{-1}T\right)^k &= 0, \\ \lim_{k\to\infty}\sum_{i=1}^{k}\left(S^{-1}T\right)^{i-1} &= \left(I - S^{-1}T\right)^{-1}.\end{aligned} \tag{32}$$

Now (32) can be used to simplify (31) when k goes to infinity, then the resulting equation is

$$\begin{aligned}\lim_{k\to\infty}\mathrm{Cov}(e_k) &= \left(I - S^{-1}T\right)^{-1} S^{-1} \\ &\qquad \times \mathrm{E}\left[(b-\tilde{b})(b-\tilde{b})^T\right] S^{-T}\left(I - S^{-1}T\right)^{-T},\end{aligned}$$

and using (29) it can be concluded that

$$\lim_{k\to\infty} \mathrm{Cov}(e_k) = K^{-1}\mathrm{E}\left[(b-\tilde{b})(b-\tilde{b})^T\right] K^{-T}.$$

Finally, by substituting $\tilde{b}$ from (28) and using (16) the following equation is obtained

$$\begin{aligned}\lim_{k\to\infty} \mathrm{Cov}(e_k) = &\left(I - K^{-1}LC\right)\\ &\times K^{-1}QK^{-T}\left(I - K^{-1}LC\right)^T.\end{aligned}$$

By comparison the computed covariance with (17), it is obvious that they are equal, thus $\tilde{b}$ which can minimize variance of the error is identical to $\hat{b}$ in (11). The main problem is existing K^{-1} in (11) which can be addressed using the following theorem.

Theorem 2. *Let*

$$\begin{aligned}\tilde{b} &= \bar{b} + Z_{ss}\left(Z_{ss}^T Z_{ss}\right)^{-1}\left(y - Z_{ss}^T\bar{b}\right),\\ Z_k &= S^{-T}T^T Z_{k-1} + S^{-T}C^T\end{aligned} \tag{33}$$

where Z_{ss} is the steady state of Z_k, then $\tilde{b}$ is identical to $\hat{b}$ given by (11).

Proof. Let Z be defined as

$$Z = K^{-T}C^T, \tag{34}$$

then by substituting Z into (11), the resulting equation is

$$\tilde{b} = \bar{b} + Z\left(Z^TZ\right)^{-1}\left(y - Z^T\bar{b}\right). \tag{35}$$

By pre-multiplication K^T into (34) and using $K = S - T$, one can get

$$Z = S^{-T}T^TZ + S^{-T}C^T. \tag{36}$$

If all eigenvalues of $S^{-T}T^T$ are strictly inside the unite circle, (35) and (36) can be considered as the steady state value of (33).

The matrix $S^{-T}T^T$ is given by

$$S^{-T}T^T = I - S^{-T}K^T. \tag{37}$$

The transpose of (29) is

$$I - K^TS^{-T} = M^{-T}\Lambda M^T \tag{38}$$

where M is the modal matrix and Λ is a diagonal matrix, the diagonal elements of which are the eigenvalues of $S^{-1}T$. If (38) be pre- and post-multiplied by S^{-T} and S^{T} respectively, then

$$I - S^{-T}K^{T} = \left(M^{T}S^{T}\right)^{-1}\Lambda M^{T}S^{T}. \tag{39}$$

Now, given (37) and (39), it can be concluded that the eigenvalues of $S^{-T}T^{T}$ and $S^{-1}T$ are equal. Furthermore, according to Lemma 1, the matrix S is selected such that all eigenvalues of $S^{-1}T$ are strictly inside the unite circle which completes the proof. □

Considering the presented Lemma 1 and Theorem 2, the developed estimation approach using (26) and (33) does not need K^{-1} and is optimal under unknown covariance of b. Therefore, our proposed approach is computationally efficient.

5. Simulation

In this section, a comparison among the proposed RCKF algorithm, CKF algorithm and one existing approach is done. The existing approach is steepest gradient descent algorithm (SGD) [16], in which a cost function is defined for the brain surface estimation error with constraint on brain model, then it is minimized using steepest gradient descent method.

To solve model's equation and simulation, one needs a software which is able to solve PDEs. A well-known software in this field is *COMSOL Multiphisycs* which solves PDE equations as FEM in one, two and three dimensional. Furthermore, A computer with Intel Quad Core i5 with 4 GB of ram running Windows 7 64bit is used to simulate deformation and estimate it.

Accuracy of linear elastic model relies on selection of Young's modulus and Poisson's ratio and the following values are used for them respectively [27]

$$E = 3kPa$$
$$\nu = 0.45$$

In this simulation, brain's physical model is assumed as a sphere 22cm in diameter which is approximately the same as the men's brain and has been used in researches [11, 32].

5.1. Boundary Conditions

To solve PDE equations, it is needed to segment the brain and define appropriate boundary condition of each segment. The segments are depicted in Fig. 1 and boundary condition of each segment is considered as follow:

part 1. This part expresses the brainstem and is considered fixed position.

part 2. This part involves middle region of the brain and is considered to move along the skull. Consequently, brain tissues can not exit from the skull.

part 3. This part is the region under surgery and can move freely.

5.2. Mesh

To solve the problem numerically, volume of the body should be subdivided to smaller geometries, called elements. The subdivision of the body is called mesh that in three dimensional can be divided to categories like tetrahedral, hexahedral and mixed-element meshes. There are many researches on the effects of different types of elements in a mesh on the accuracy of solutions [33], and results express that the appropriate meshes for medical filed is tetrahedral meshes. The Fig. 2 depicts tetrahedral meshes of the used sphere.

5.3. Simulation Results

After the preceding steps, it is possible to simulate the deformation. The Fig. 3 expresses simulation results and Fig. 4 depicts different cross sections of sphere and field of deformation.

Now the deformation is simulated, in order to estimate that the estimation approaches can be used. To this end, it is assumed that the initial estimation is zero, i.e. $\bar{b} = 0$. The Fig. 5 depicts actual and estimated deformation computed by RCKF. In FEM, elements are interconnected at points called nodes. Each unique node, has its own unique number, and the *Node number* in Fig. 5 refers to these numbers.

In order to show that the result of RCKF tends to the estimation of CKF, the norm of estimation error of RCKF per iteration is indicated in solid line, and that of CKF in dashed line in Fig. 6. It can be seen that RCKF has resulted in a more accurate estimation after 37 iterations ($Iter_{min}$). This difference in the accuracy is due to round-off error, which has occurred in the inverting K.

As illustrated in Fig. 7, the maximum of error in the proposed RCKF and CKF is lower than the maximum error in SGD. In order to draw a comparison, specifications of estimation error along with computation time of estimation approaches are reported in Table 1. To have a more accurate computation time, each estimation is computed 10 times and average of intra-operative computation times are listed in the Table 1.

It can be inferred from the Table 1 that RCKF is superior in terms of accuracy, and has the minimum intra-operative computation time. RCKF, therefore, is computationally effective and accurate.

6. Conclusions

In this paper, brain surface imaging together with CKF was employed to improve the responses of linear elastic model which can be utilized to increase the accuracy of neuronavigation systems. Furthermore, it was shown that the computed estimation is optimal under some conditions. Moreover, a recursive approach (RCKF) was proposed, which does not require to calculate inverse of a large matrix. Finally, it was shown that RCKF is computationally effective as well as optimal.

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Table 1: Summary of the norm and the maximum of estimation error (norm(e) and max(e)) and intra-operative computation time (t_{intra}) of estimation approaches.

Estimation approach	$t_{intra}(s)$	norm(e) (m)	max(e) (m)
RCKF	40	8.1×10^{-3}	3.71×10^{-4}
CKF	61.8	8.2×10^{-3}	3.81×10^{-4}
SGD	177	25×10^{-3}	9.31×10^{-4}

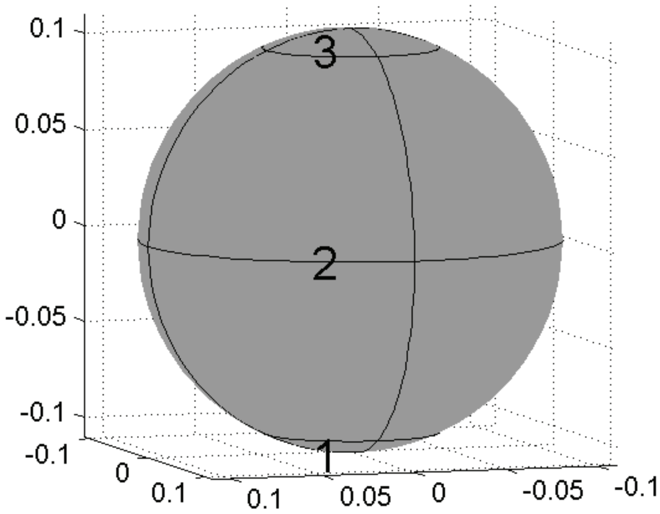

Figure 1: The defined boundary conditions of model. Surface 1 is fixed, surface 2 can move along the skull but not along the normal direction, and surface 3 is free.

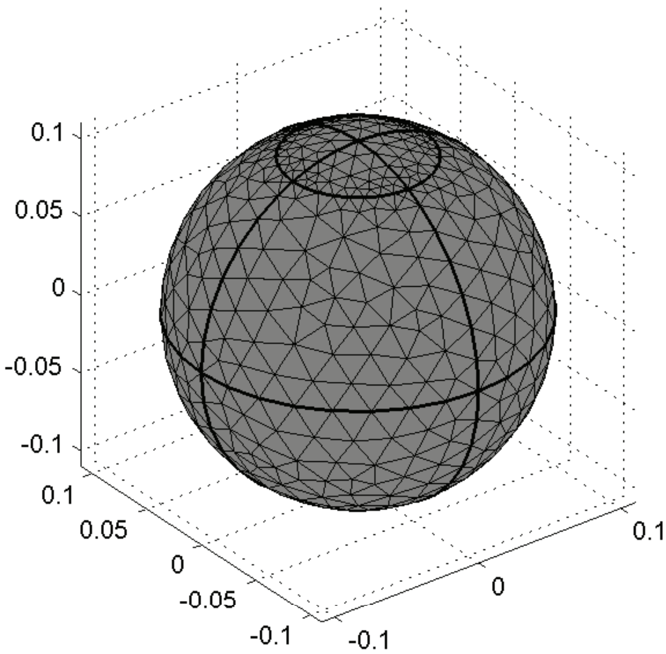

Figure 2: The result of meshing the volume.

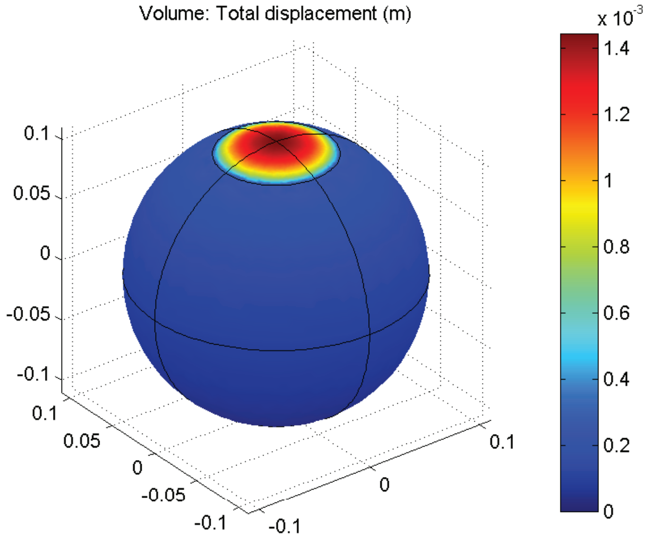

Figure 3: The result of simulation of deformation using linear elastic model.

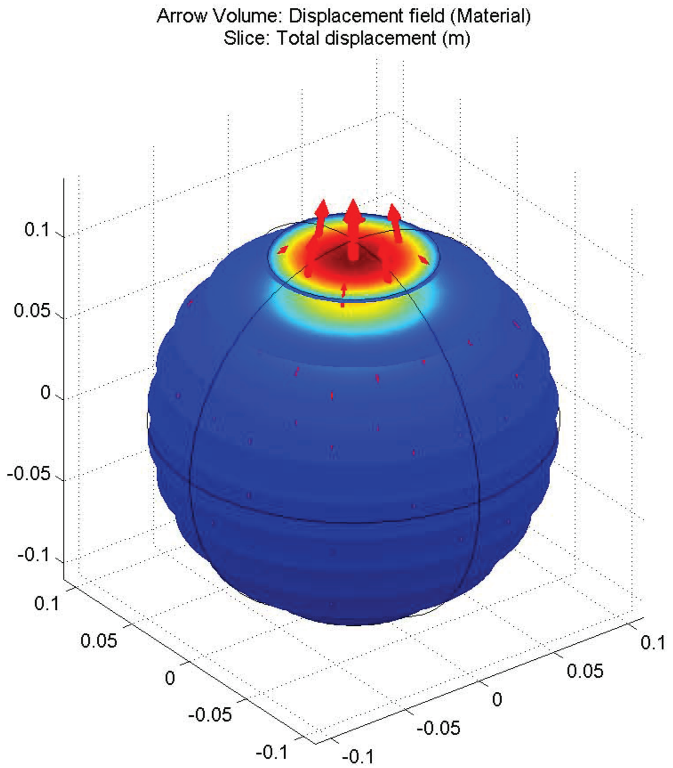

Figure 4: The slice plot and filed of deformation.

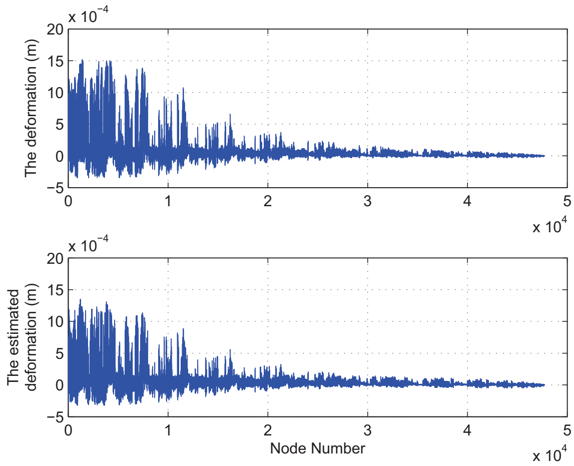

Figure 5: The real deformation and estimation of RCKF.

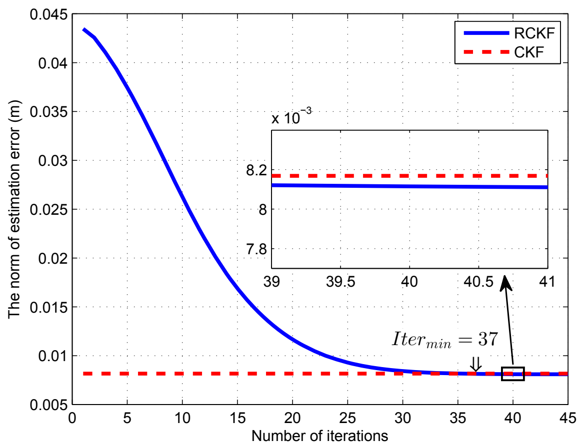

Figure 6: The norm of estimation error of CKF (dashed line) and RCKF per iteration (solid line).

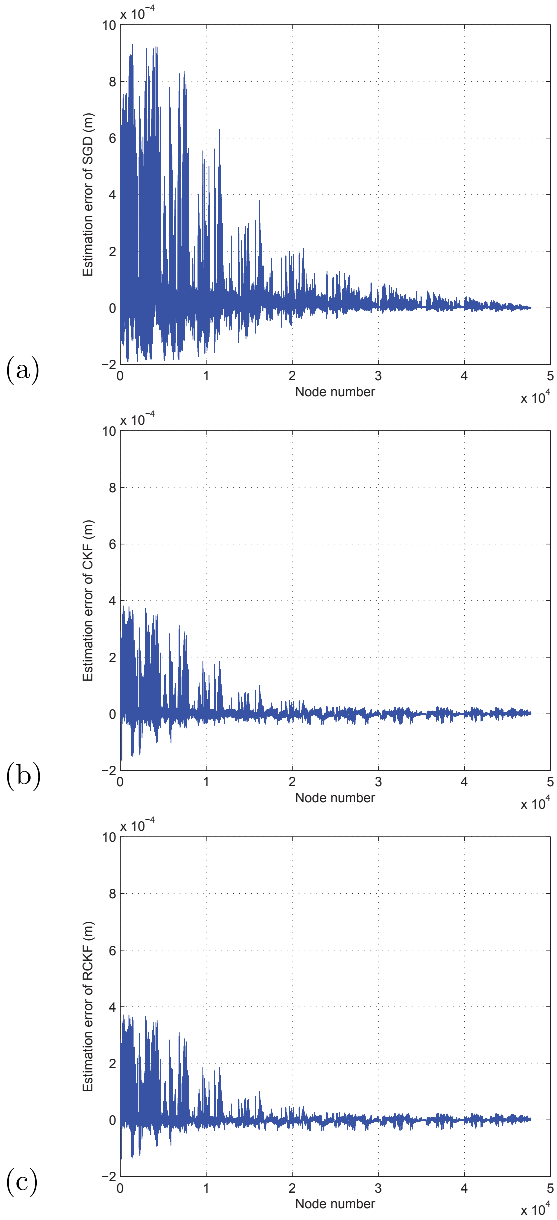

Figure 7: Estimation error of SGD (a), CKF (b), and RCKF (c).